International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 3, Issue 3, May 2014, 31-38

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

VECTOR OPTIMIZATION PROBLEM AND ITS APPLICATIONS IN SYSTEM RELIABILITY

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ABSTRACT

The paper proposes an algorithm for `Hybrid Approach' to solve Vector Optimization Problem (VOP) and the method is useful for convex and non convex optimization problems. Using this approach we test the reliability of the system design. By means of test problems, we illustrate the strengths and advantages of the approach over existing results.

KEYWORDS: Vector Optimization, Scalarization, Pareto Front, Numerical Methods, System Reliability

AMS Subject Classifications: 90C26, 90C29, 90C30, 90C56

1. INTRODUCTION

The world today has become a challenge with the influence of the innovations of modern science. People have to take many decisions constantly. Whenever a decision is taken rationally, the possible alternatives have to be considered to select the best one. The judgment process requires understanding of what is better or worse alternative taking into account a performance criterion. In the real world, many problems have been modelled as optimization problems whose objective function involves a certain number of reasonable performance criteria. We have to choose at least one individual better off, without making any other individual worse off. These types of problem we call vector optimization problem (VOP) (see [1]-[4]).

To optimize all such objective functions at the same time is not a simple task. Hence the intent of VOP is to figure out the best possible trade-off among these objectives. A set of solutions is found while a trade-off is conducted among these conflicting objective functions. The set of all such optimal solution is called Pareto optimal solution and the line which consist the images of these optimal solutions is called Pareto front. A common approach to solve VOP is reformulating the problem into the scalarization problem with some parameters. Then solve the problem by choosing an appropriate method that is used for single objective optimization problem. There are many ways to solve VOP and the most conventional way is to characterize efficient points in terms of optimal solutions of appropriate scalar optimization problems (see [5]-[7]). Among of many possible ways of obtaining a scalar problem from VOP, the following are common: weighted sum method [8,9], epsilon constraint method [5] and NSGA II [10]. Nowadays, by using computer technology it is possible to approximate the whole Pareto front by the variation of parameters. And it is important for many decision makers to see the all the alternative available solutions which headed to their goal.

Multiobjective optimization problem has many real-life applications which arise in a wide range of areas, such as engineering, economics, game theory, management science, Internet and environmental control [1, 11, 12]. In many of these applications, methods of traditional single-objective optimization are not enough; new techniques and concepts are needed.

Thus our intent is to choose a suitable and most reliable method to solve VOP, and this can allow us to get better result in the system design. In our analysis we used the technique, which is called `Hybrid Approach', introduced in [13, 14]. We choose this technique since it is very efficient to generate the whole Pareto front. Also this method is very useful when problems include convex and non convex functions.

2. PRELIMINARIES

We recall now some basic notation and tools of vector optimization problems. Let $\mathbf{R^n}$ be the n-dimensional Euclidean space. Compare $x, y \in \mathbf{R^n}$, as follows [15].

$$x \geqq y, \text{ iff } x_i \geqq y_i, \; i = 1,2,\ldots,n.$$

For $x, y \in \mathbf{R^n}$, we say that

$$x \geqq y \text{ iff } (\mathrm{x_i} - y_i) \text{ is non-negative,}$$

$$x \geq y, \text{ iff } x \geqq y \text{ and } x \neq y,$$

$$x > y, \text{ iff } x_i > y_i \text{ and } i = 1,2,\ldots,n.$$

For $l, m \in \mathbf{N^n}$, we consider the following vector optimization Problem (P):

$$(P) \quad \begin{cases} \min \; f(x) := [f_1(x),\ldots,f_l(x)] \\ s.t. \;\; x \in X := \{x \in R^n \mid g_j(x) \leqq 0, \; j = 1,\ldots,m\} \end{cases} \tag{1}$$

where the functions $f_i : \mathbf{R}^n \to \mathbf{R}, \; i = 1,\ldots,l,$ and $g_j : \mathbf{R}^n \to \mathbf{R}, \; j = 1,\ldots,m,$ are continuous and real-valued functions. Note that the functions f_i and g_j are convex and differentiable.

The set of solutions of (P) is known as efficient points [3], or Pareto points [1], and next we recall the definition of efficiency of VOP.

Definition 2.1: A point $\bar{x}$ is said to be *efficient* for Problem (P) iff there is no $x \in X$ such that $f(x) \leq f(\bar{x})$. We denote the set of efficient points of Problem(P) as E(P).

Define the set of positive weights,

$$W := \left\{ w \in \mathbf{R}^l \,\middle|\, w_i > 0, \; \sum_{i=1}^{l} w_i = 1 \right\}.$$

In our study, we focus on the following existing parameter-based scalarization approach.

The Hybrid Approach: A parameter-based scalarization approach is presented by Corely (1980) and Wendell and Lee (1977) in slight different forms. For a given $w \in W$, the scalarization is stated as follows.

$$(HS)\quad \begin{cases} \min\limits_{x\in X} & \sum\limits_{i=1}^{l} w_i f_i(x) \\ \text{subject to} & f_i(x) \leqq \alpha_i, \quad \text{for all } i=1,\dots,l, \end{cases} \tag{2}$$

where $\alpha_i = f_i(\bar{x})$ and $\bar{x} \in X$.

Remark 2.1: The advantage of this method is that the method combines the positive features of the weighting method [8, 9] and the ε-constraints method [5]. Also, by changing the weights, uniformly distributed Pareto points are to be generated.

2.1 Application to System Reliability

Optimization technique is often used to develop system design. In this case, the system reliability is maximized subject to resource constraints. To achieve the best system design, it is often desirable to simultaneously maximize system reliability and minimize resource consumption. Since the problem involve more than one objective, therefore, it is better to apply VOP to develop system design.

The reliability optimization problem is formulated as follows.

$$(ROP)\quad \begin{cases} \max\left(\prod\limits_{i=1}^{s} R_i(x_i)\right), \ \min\left(\sum\limits_{i=1}^{s}\sum\limits_{j=1}^{m_i} c_{ij}x_{ij}\right), \min\left(\sum\limits_{i=1}^{s}\sum\limits_{j=1}^{m_i} w_{ij}x_{ij}\right), \\ \text{subject to } 1 \leq \sum\limits_{j=1}^{m_i} x_{ij} \leq n_{\max,i}, \quad \text{for all } i=1,\dots,s, \\ \text{and } x_{ij} \in \{0,1.2,\dots\}. \end{cases} \tag{3}$$

Here, the notation R is system level reliability, c_{ij} and w_{ij} are the cost and weight for the j^{th} available component for subsystem i respectively. Moreover, $n_{\max,i}$ is maximum number of components in parallel used in subsystem i, and x_{ij} is a quantity of the j^{th} variable component used in subsystem i (see for more information in [16]).

3. NUMERICAL EXPERIMENTS

In this section, extensive numerical experiments are conducted. Here we use MATLAB for coding and `fmincon' solver is used to solve the problems. The algorithm of the Problem (HS) is as follows.

Algorithm HS (Hybrid Scalarization for $l=3$)

Step 1 (Input)

Set the number of partition points (N+1) in the interval of weights. Set $s=0$.

Step 2 (Determine the Boundary Points of the Pareto Front)

- Find $\bar{x}_0$ that solves Problem (P_3):= $\min\limits_{x\in X} f_3$.

  Let $\bar{f}_1 := f_1(\bar{x}_0)$, $\bar{f}_2 := f_2(\bar{x}_0)$ and $f_3^* := f_3(\bar{x}_0)$.

Let $w_1 := f_3^* f_2 / \left[\bar{f}_1 + \bar{f}_2 + f_3^*\right]$.

Set $F(s) := \left[f_1(\bar{x}_0), f_2(\bar{x}_0), f_3(\bar{x}_0)\right]$.

- Similarly generate w_2 and w_3.

Step 3 (Generate a Weight Partition)

For $i_1 := 0,1,...,$ N.

$\{$

For $i_2 := 0,1,...,\ i_1$.

$\{$

Let

$w_1 := (N - i_1)/N,$

and $w_2 := (i_1 - i_2)/N,$

$\}$

$\}$.

Record w_1, w_2 and $w_3 := 1 - w_1 - w_2$.

Step 4 (Select $\hat{x}$ and Solve the Scalarized Problems (HS)): Select any feasible point $\hat{x}$ and w_i, i=1,2,3 are provided into the Problem (HS). Then solve (HS) and results are recorded.

3.1 Example

We adapt the following test problem from [16], and the multi-objective problem with three objectives, which consists maximize system reliability, minimize total cost and minimize system weight.

$$\min \quad (r_1 r_2 r_3,\ c_1 + c_2 + c_3, w_1 + w_2 + w_3),$$

where

$$r_1 := -1 + \left\{(1-0.94)^{x_1}(1-0.91)^{x_2}(1-0.89)^{x_3}(1-0.75)^{x_4}(1-0.72)^{x_5}\right\},$$

$$r_2 := -1 + \left\{(1-0.97)^{x_6}(1-0.86)^{x_7}(1-0.70)^{x_8}(1-0.66)^{x_9}\right\},$$

$$r_3 := -1 + \left\{(1-0.96)^{x_{10}}(1-0.89)^{x_{11}}(1-0.72)^{x_{12}}(1-0.71)^{x_{13}}(1-0.67)^{x_{14}}\right\},$$

$$c_1 := 9x_1 + 6x_2 + 6x_3 + 3x_4 + 2x_5,$$

$$c_2 := 12x_6 + 3x_7 + 2x_8 + 2x_9,$$

$$c_3 := 10x_{10} + 6x_{11} + 4x_{12} + 3x_{13} + 2x_{14},$$

$$w_1 := 9x_1 + 6x_2 + 4x_3 + 7x_4 + 8x_5,$$

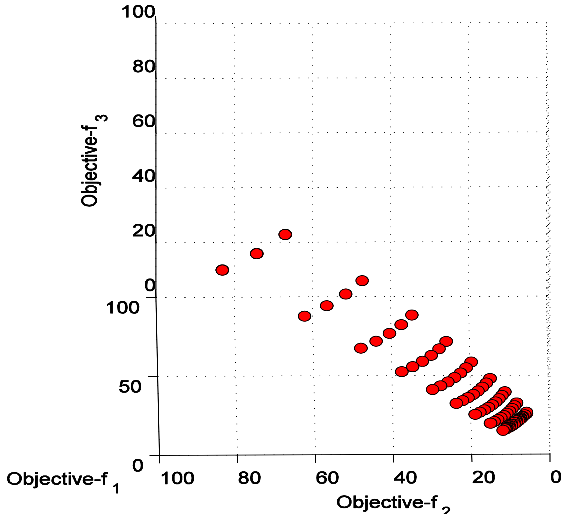
$$w_2 := 5x_6 + 7x_7 + 3x_8 + 4x_9,$$

$$w_3 := 6x_{10} + 8x_{11} + 2x_{12} + 4x_{13} + 4x_{14},$$

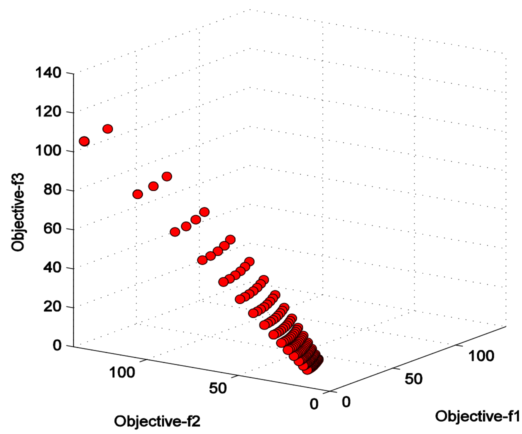
s.t.

$$1 \le \sum_{i=1}^{5} x_i \le 8,\ 1 \le \sum_{i=6}^{9} x_i \le 8,\ \text{and}\ 1 \le \sum_{i=10}^{14} x_i \le 8.$$

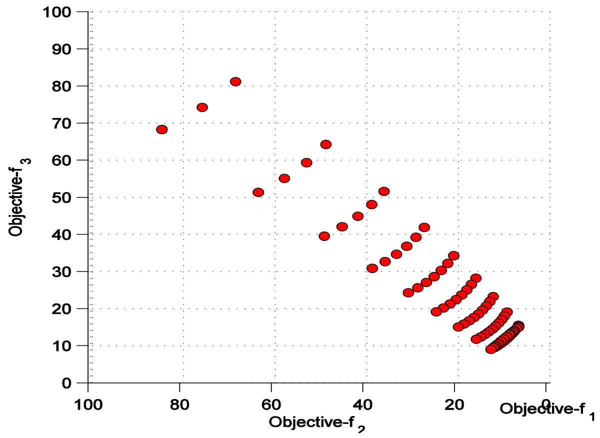
Uniformly distributed weights are provided and the resulting Pareto points are shown in Figure 1. The Pareto points are obtained and their distribution is uniform for the weighted-sum approach. The method obtained 103 Pareto points in the Pareto front.



(a) Points Found by Algorithm HS



(b) Points Found by Algorithm HS



(c) Points Found by Algorithm HS

Figure 1: Pareto Points Found for the System Reliability with N = 10

4. CONCLUSIONS

We have proposed a new technique and an algorithm for generating an approximation of the Pareto front of system reliability problems, in particular problems with a convex and non-convex Pareto front. The numerical experiments we conducted with such problems suggest that our proposed technique, using Hybrid Method in the system reliability, is more successful compared to the existing vector optimization techniques for system reliability which are available in the literature. The new technique seems to be particularly useful when it is mandatory to approximate the Pareto front so that decision maker will have enough options to take best suitable strategies for improving system reliability, and this was demonstrated in the Example 3.1

It would also be interesting to apply the new technique to non-linear system control real life problems as an extension to the work done in here.

REFERENCES

1. Miettinen, K. M.: Nonlinear Multiobjective Optimization. Kluwer Academic, Boston (1999).
2. Quagliarella, D., Vicini, A: Sub-population policies for a parallel multiobjective genetic algorithm with applications to wing design. Proc. IEEE Int. Conf. Syste., Man, and Cybern. 4, 3142-3147 (1998).
3. Geoffrion, A. M.: Proper efficiency and the theory of vector maximization. J. Math. Anal. Appl. 22, 618-630 (1968).
4. Audet, C., Savard, G., Zghal, W.: A mesh adaptive direct search algorithm for multiobjective optimization. Euro. J. Oper. Res. 204, 545-556 (2010).
5. Chankong, V., Haimes, Y. Y.: Multiobjective Decision Making: Theory and Methodology. North-Holland, Amsterdam (1983).
6. Eichfelder, G.: Adaptive Scalarization Methods in Multiobjective Optimization. Springer, Berlin, Heidelberg (2008).
7. Eichfelder, G.: An Adaptive scalarization method in multi-objective optimization. SIAM J. Opt. 19, 1694-1718 (2009).

8. Gass, S., Saaty, T.: The computational algorithm for the parametric objective function. Naval Res. Logis. Quart. 2, 39-45 (1955).

9. Zadeh L.: Optimality and non-scalar-valued performance criteria. IEEE Transactions on Automatic Control 8, 59-60 (1963).

10. Deb, K., Pratap, A., Agarwal, S., Meyarivan, T.: A fast and elitist multiobjective genetic algorithm: NSGA-II. IEEE Trans. Evol. Computat. 6, 182-197 (2002).

11. Annamdas, K. K., and Rao, S. S.: Multi-objective optimization of engineering systems using game theory and particle swarm optimization, Engineering Optimization, 41(8), 737-752 (2009).

12. Chinchuluun, A., Pardalos, P. M. and Migdalas, A.: Pareto Optimality, Game Theory and Equilibria, Springer Optimization and its Applications, 17, (2008).

13. Corley, H. W.: A new scalar equivalence for Pareto optimization, IEEE Transaction on Automatic Control, 25 (4), 829-830 (1980).

14. Wendell, R. E., Lee, D. N., Efficiency in multiple objective optimization problems, Mathematical Programming, 12, 406-414 (1977).

15. Mangasarian, O. L.: Nonlinear Programming. SIAM, Philadelphia (1994).

16. Taboada, H., Coit, D. W.: Data clustering of solutions for mul-tiple objective system reliability optimization problems. Quality Tech-nology and Quantitative Management Journal, 4(2), 35-54 (2007).